## Dynamic Analysis of A Two-link Flexible Manipulator Incorporating Payload

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## Abstract

The paper describes the dynamic modelling of a flexible manipulator with two flexible links. This study rotates in the horizontal plane. Based on a combined Euler-Lagrange formulation and assumed mode method to formulate the dynamic model of a flexible manipulator system. A payload is added at the tip of the second link. Links are modelled as Euler-Bernoulli beams of uniform density with clamped-mass boundary conditions. The final dynamic equation can be obtained by computing the Lagrangian and then using Lagrange's equation of motion for each joint and node. For a two-link flexible manipulator, the effect of structural damping in the dynamic model is examined and analyzed. Simulation results are analyzed in the time to assess the accuracy of the model in representing the actual system. The model equations are verified with various payload profiles.

## 1. Introduction

A two-link flexible manipulator is attractive because they avoid the large inertia forces associated with traditional, large-section, rigid-link manipulators. Flexible robot manipulators require less material, are lighter in weight, consume less power, require smaller actuators, are more maneuverable and transportable, have less overall cost and higher payload to robot weight ratio. Unlike rigid manipulators, the dynamics of manipulators incorporate the effects of mechanical flexibilities in its links.

Link flexibility is a consequence of the lightweight constructional feature in the manipulator arms that are designed to operate at high speed with low inertia. Thus, flexible manipulators undergo two types of motion, i.e. rigid and flexible motion. Because of the interaction of these motions, the resulting dynamic equations of flexible manipulators are highly complex and, in turn, the control task becomes more challenging compared to that for rigid robots. Each flexible link can be modeled as distributed parameter system where the motion is described by a coupled system of ordinary and partial differential equations (PDE).

PDEs and boundary equations of a two-link flexible manipulator system are obtained by matching the shear force and bending moment at the elbow joint, allowing the eigenvalues to be computed without recourse to dynamic formulations [1]. On the other hand, the vibration modes of a generic two-link flexible manipulator are studied as a function of the link, rotor and tip mass distribution. Necessary and sufficient conditions are developed for all vibration modes to exhibit a node at the manipulator.

Combaining Euler-Lagrange formulation and asumed mode method (AMM) approach to model the planar motion of a manipulator consisting of two flexible links and joints. The conventional Lagrangian modeling of flexible link robots does not fully incorporate the bending mechanism of flexible link as it allows free link elongation in addition to link deflection by Subudhi and Morries [2]. Using the AMM to derive a dynamic model of multilink flexible robot arms limiting to the case of planar manipulators with no torsional effects by De Luca and Siciliano [3]. The equations of motion which can be arranged in a computationally efficient closed form that is also linear with respect to a suitable set of constant mechanical parameters have been obtained [4]. Various input torque profiles used to investigate the dynamic model

The flexibility of the links is distributed along their length, necessitating that they be regarded as deformable bodies that have an infinite number of degrees of freedom. However, for practical purposes, the system of flexible links must be reduced to a system having a finite number of degrees of freedom, and the AMM is an appropriate technique for achieving this. This involves the synthesis of a finite dimensional dynamics model of the flexible system from the infinite dimensional model resulting from the solution of partial merentid equations. Each flexible link can be modelled as distributed parameter system